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INVESTIGATION OF THE AXIALLY SYMMETRIC FLOW OF A FRICTIONLESS FLUID THROUGH TURBOMACHINES

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Introduction — The equation of motion of a frictionless fluid in the relative coordinate system admits of drawing some new general conclusions. By using the energy equation, we obtain the conditions which must be satisfied by the force acting on the particle of fluid.

When these conclusions are applied to the axially symmetric flow through turbomachines, it becomes evident that the stream-lines lie always on the blade surfaces. Therefore, no further conditions are necessary for the stream-lines to lie on the blade surfaces.

General formulation — In the relative coordinate system moving with the velocity $\vec{u}$ the equation of motion may be [1][1]

$$(1) \qquad \vec{F} = \frac{1}{\rho} \operatorname{grad} p + \operatorname{grad}\left(\frac{c^2}{2} - \vec{u}\cdot\vec{c}\right) - (\vec{c} - \vec{u}) \times \operatorname{rot} \vec{c} + \frac{\partial'}{\partial}\frac{c}{t}.$$

In this equation, $\vec{F}$ is a force acting on the particle of fluid, p is the pressure, ρ the density, $\vec{c}$ the absolute velocity, and the line in the time derivative emphasizes that the differentiating is done in the relative coordinate system.

For the steady flow of a barotropic fluid $[\rho = f(p)]$ the equation (1) becomes

$$(2) \qquad \vec{F} = \operatorname{grad}\left(\int \frac{dp}{\rho} + \frac{c^2}{2} - \vec{u}\cdot\vec{c}\right) - (\vec{c} - \vec{u}) \times \operatorname{rot} \vec{c}.$$

[1] Numbers in square brackets refer to the references at the end of the paper.

The energy equation for turbomachines has the form [2]

$$(3) \qquad \int \frac{dp}{\rho} + \frac{c^2}{2} - \vec{u} \cdot \vec{c} = H,$$

where H is constant along any particular stream-line in the area of one row of vanes, but may vary when passing from one strem-line to another. Now, we shall do the analysis using the equations (2) and (3).

If the equation (3) is satisfied, the equation (2) becomes

$$(4) \qquad \vec{F} = \text{grad } H - (\vec{c} - \vec{u}) \times \text{rot } \vec{c}.$$

There are three cases to consider here:

1. Let $H =$ const. the absolute constant and the velocity be potential, rot $\vec{c} = 0$, then $\vec{F} = 0$. There must be no external forces acting on the fluid.

2. If H varies and the velocity be potential, rot $\vec{c} = 0$, or rot $\vec{c}$ is parallel to the relative velocity $\vec{w} = \vec{c} - \vec{u}$, we obtain

$$(5) \qquad \vec{F} = \text{grad } H;$$

the extraneous forces have a potential.

3. In turbomachines H is constant along any particular sream line, but may vary as we pass from one stream-line to another, therefore

$$(6) \qquad \vec{ds_r} \cdot \text{grad } H = 0,$$

where $\vec{ds_r}$ is the infinitely small path of the fluid particle in relation to the relative coordinate system.

The scalar product of the equation (4) with $\vec{ds_r} = \vec{w}\, dt = (\vec{c} - \vec{u})\, dt$ gives

$$\vec{F} \cdot \vec{ds_r} = \vec{ds_r} \cdot \text{grad } H - \vec{ds_r} \cdot [(\vec{c} - \vec{u}) \; x \; \text{rot } \vec{c}].$$

the first term on the right-hand side of this equation is equal to zero because of (6), and the second term is zero since $\vec{c} - \vec{u}$ is parallel to $\vec{ds_r}$. So we have

$$(7) \qquad \vec{F} \cdot \vec{ds_r} = 0, \text{ or } \vec{F} \cdot \vec{w} = 0.$$

The force $\vec{F}$ is perpendicular to the relative velocity w.

Axially symmetric flow through turbomachines — When there are many vanes, we may assume that the flow is axially symmetric. This means that the blade row has an infinite number of infinitely thin blades. The local blade force is normal to the local blade surface, so we have

$$(8) \qquad \vec{F} \cdot \vec{ds} = 0,$$

where $\vec{ds}$ is an infinitely small path in the tangental plane of the blade.

Comparing (7) with (8) we conclude that $\vec{ds}$ and $\vec{ds}_r$ are normal to the force $\vec{F}$, so $\vec{ds}$ and $\vec{ds}_r$ must be in the same plane. The stream-lines lie on the blade surfaces.

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