“Non-Reflective” Boundary Design via Remote Sensing and PID Control Valve

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Abstract: In this paper, “Non-reflective” (or “semi-reflective”) boundary condition is established using the combination of a remote sensor and a control system to operate a modulated relief valve. In essence, the idea is to sense the pressure change at a remote location and then to use the measured data to adjust the opening of an active control valve at the end of the line so as to eliminate or attenuate the wave reflections at the valve and thus to control system transient pressures. This novel idea is explored here initially by the means of numerical simulation and shows considerable potential for transient protection, as demonstrated in examples. Using this model, wave reflections and resonance can be effectively eliminated for frictionless pipelines or initially no-flow conditions and better controlled in more realistic pipelines for a range of transient disturbances. In addition, the feature of even order of harmonics as well as “non-reflective” boundary conditions during steady oscillation, obtained through time domain transient analysis, are verified by hydraulic impedance analysis in the frequency domain.

CE Database subject headings: Non-reflective boundary, PID control valve, remote sensor, wave reflection, resonance, time domain analysis, frequency domain analysis.

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1. Introduction to “Non-Reflective” Boundaries

One of the most interesting aspects of transient phenomenon is that they are primarily created and controlled by the action of boundary devices in the system. The actions of, or changes to, these boundaries both initiate the transient event and largely control its severity, creating by these actions a sequence of velocity and pressure fluctuations that then propagate throughout the pipe system. The action or response of other devices and components, either by design or by their intrinsic nature, then reflects, refracts, amplifies or attenuates the primary pressure and velocity waves.

There is an intriguing connection between waterhammer control and the use of “tailored” or active boundary conditions that control wave reflections. Certainly such “non-reflective” boundary conditions have been partially explored before and used in the past to represent certain network junctions and components (Almeida and Koelle 1992; Pejović-Milić et al. 2003). Indeed in a conventional sense, various “semi-reflective” boundaries are the basis of waterhammer protection, and their role as general energy dissipation device in systems with dead-ends is extended and explained in the following sections; however, as some devices can be sources of resonance, special consideration must be given to the frequency domain. Moreover, the physical, mathematical and numerical aspects of such boundaries are considered and developed along with possible applications that move these boundary conditions toward use and application in systems and devices.

However, to be effective the boundary actions must be controlled and a wider range of approaches is now possible. In particular, with the development of electronic
and computer technologies, dynamic PID (proportional, integral and derivative) controllers are now being used more frequently to maintain a desired performance in a water system; the variable to be controlled may be turbine speed, turbine power output, pump speed, pump torque, pump discharge flow, water level in tanks, upstream or downstream pressure at valve, and so on. This developed control capability also provides an alternative arrangement for suppression of hydraulic transients in pipe systems. In fact, local/conventional PID control valves, even when modulating, have implications for transient control by maintaining a desired pressure or flow in the system. However, the combination of remote sensing and PID control valve could provide a broad range of waterhammer protection through designing of “non-reflective” boundaries into a pipe system. Yet, the relevant literature to date has mainly focused on the responses of transient flow to the action of PID control valves by coupling hydraulic transient analysis and control theory (Koelle 1992; Koelle and Poll 1992; Lauria and Koelle 1996; Bounce and Morelli 1999; and Poll 2002), it is an innovation to study how to actively control a valve in response to the remotely sensed pressure for system waterhammer protection.

The goal of this study is to explore how “non-reflective” boundaries can be designed and applied for transient protection, particularly at problematic locations like dead-ends or cul-de-sacs in a distribution system. Though having the same role to limit the transient pressure in pipe systems, this approach is different from the operation of a pressure relief valve (PRV) that opens a by-pass line to release excessive flow when the pressure in pipeline exceeds the set point. Such as an arrangement is also quite distinct from a local/conventional pressure sustaining valve or backpressure valve (holding the
pressure at valve inlet or outlet) that modulates the valve opening to maintain the set point corresponding to the locally sensed pressure (Hopkins 1998). The key issue in the remote control is the transformation of transient pressure waves between the remote sensor and active control valve. Since a fully non-reflective valve is likely to be mostly a theoretical concept and will be difficult to fully achieve in practice (though we maintain the idea is still conceptually beneficial), we use in this paper the term “non-reflective” (with quotation marks) to continually remind the reader of this theory-reality tension.

In this paper, an example is first presented to demonstrate the possibility of dangerous waterhammer occurring in the system due to reflections at dead-ends. The role of a control valve to dissipate the transient energy and thus protect the system from excessive transient pressures is illustrated. Then, the mathematical model for the local/conventional PID control valves is addressed as a prerequisite to the solution of remote sensing and “non-reflective” valve opening, and the key novel features in the remote control model are discussed. After that, examples involving a successful numerical application of the remote control model are presented. Theses examples show the ability of “non-reflective” boundary to control the reflection of pressure wave and potential resonance within the pipeline. Moreover, the developed “non-reflective” boundary condition during the steady oscillation is verified using hydraulic impedance analysis in frequency domain. Finally, using the developed simulation tool, the selection and tuning of PID controller parameters are discussed based on sensitivity analyses.
2. Transient Performances with Dead-ends and Valve Control

A dead-end is often a tricky arrangement in a pipe system. Such an arrangement includes a closed valve, which carries no discharge and can cause unexpected high pressure in the system. When a pressure wave is transmitted into the dead-end pipe, no flow or further wave transmission is physically permitted at the line’s termination, which induces a doubling in the local pressure and creates a reflected pressure wave that returns into the system. This is the so-called dead-end reflection. By contrast, a pressure wave would be fully reflected with reverse sign from a constant head reservoir. Interestingly, then, neither a reservoir nor a dead-end is intrinsically dissipative; they may reflect waves but they conserve the transient energy. By contrast, a partially open valve, say at a reservoir, dissipates energy and acts between a dead-end and a reservoir, typically reflecting some of the pressure and some of the flow. If the size of the valve opening is systematically adjusted, a value can be found for a given system so that the disturbance/excitation does not reflect at all. This setting thus produces a “non-reflective” boundary with a maximum rate of transient/oscillation energy dissipation.

Consider a pipeline system with its initial conditions described in Figure 1. If the initial pressure condition is released, two pressure waves, both 15 m in amplitude, are created and propagate into the system. The response to the traveling pulse waves are simulated and shown in Figure 2, representing the envelope of maximum and minimum transient pressures along the pipe length from A to C and then from C to F. More specifically, in Figure 2 (a), the terminal valves keep fully closed, and we see that the wave is reflected and magnified by the dead-end due to the overlapping of incident and
reflected pressure waves. In Figure 2 (b), the terminal valves now are opened to 10% of the full size in 1 second when the pulse waves start to travel, which reduces the maximum pressure but causes the unacceptable negative pressure. Figure 2 (c) represents the condition that the terminal valves are opened to 0.35% of full size in 1 second, and in this case the wave energy is largely dissipated when it arrives at this small orifice. These differences in system response can be exploited. Indeed, even when the valve opening is chosen somewhat arbitrarily, the transient pressures are likely to be at least partly mitigated. So, what if the valve opening is systematically refined to eliminate the wave reflection?

Further insight is obtained by comparing the responses for different sizes of valve opening in a single pipeline system, that is, a uniform pipeline links two reservoirs with the same constant head, which is a similar system as sketched in Figure 1. But, in this case, there are no branched pipes, and a control valve is installed at the right hand reservoir. A traveling pulse wave is initially created within the middle section as described in Boulos et al. (2006). Figure 3 shows the transient pressure head along the pipeline after reflections from both ends. The sign of wave reflection shifts as the right hand boundary shifts with the valve opening increasing from fully closed (i.e., dead-end) to fully opened (i.e., constant-head reservoir). By systematic adjustment, a valve opening of about 30% is found in this system to eliminate all wave reflections.

In practice, such “non-reflective” boundary would and could only be “tailored” by an automatic control valve, which measures and dynamically adjusts in response to the incoming pressure waves.
3. Mathematical Model for Local/Conventional PID Control Valve

PID control valves are usually installed at the connection of subsystems to maintain the desired operating condition in hydraulic networks. Most probably, the subsystems were originally designed for separate operation and then have been connected or expanded due to the urbanization and development of water distribution networks (Bounce and Morelli 1999). For instance, to ensure the minimum flow demand in downstream subsystem, an accurate and continuous control of the pressure or flow rate in the connection pipeline is needed.

The mathematical model and numerical simulation of PID control valves provides a tool to better understand the system hydraulics. Usually, there is a built-in PID controller and sensor at a control valve. For different control variables, the mathematical models are largely the same except for a slight difference in the PID controller equation.

3.1 Extended MOC Equations

To simulate a PID control valve in a pipe network (such as that in Fig. 4), the extended Method of Characteristics (MOC) is used to relate nodal heads to flow (Karney and McInnis 1992). Although the values of the characteristic constants are obviously crucial in practice, the key of the MOC equations for a fixed rectilinear grid are a linear relation between the instantaneous flow and pressure:

\[ H_1 = C'_1 - B'_1 Q \]  \hspace{1cm} (1)

\[ H_2 = C'_2 + B'_2 Q \]  \hspace{1cm} (2)
3.2 Valve Discharge Equation

Valve discharge equation defines the relationship between the flow passing through a valve and the head difference across the valve. The commonly used valve discharge equation is as follows:

\[ Q = \tau \sqrt{H_1 - H_2} \]  \hspace{1cm} (3)

in which \( E_S \) is a valve conductance parameter determined by the energy dissipation potential of the valve, and \( \tau \) is the dimensionless valve opening. For the steady flow \( Q_0 \) and the corresponding head loss of \( H_0 \), \( \tau = 1 \) and \( E_S = \frac{Q_0}{\sqrt{H_{10} - H_{20}}} \); and for no flow case with the valve closed, \( \tau = 0 \) (Wylie et al. 1993).

Instead of the predefined opening or closing motion of a conventional valve, the opening of a PID control valve is adjustable in response to the sensed pressure or pressure difference, which is desired to be controlled. That is, \( \tau(t) \) is unknown for a PID control valve and needs to be dynamically determined by the characteristics of the PID controller.

The valve relationship, equation (3), is suitable for quasi-steady flow only and might not be valid for rapid transient flow, because during transient state it may occur that the flow direction is inconsistent with the head difference across the valve when valve opening is significantly small. This phenomenon has similar influence as the backlash or dead time of the valve. Future research should consider this inconsistency in
the model, but the challenge is that there is a little data on transient behavior of valves and other components of the hydraulic system.

### 3.3 PID Controller Equations

The output signal or response of a typical parallel-structured PID controller, \( r(t) \), is given as (Tan et al. 1999),

\[
 r(t) = K_C \left( e + \frac{1}{T_i} \int_0^t e dt + T_d \frac{de}{dt} \right) \tag{4}
\]

where \( K_C \), \( T_i \) and \( T_d \) are, respectively, the proportional gain, integral time and derivative time constants of a PID-controller; they represent the characteristics of the controller. \( e \) is the controller error, that is the deviation of the process variable \( u(t) \) from its set point \( u^* \). And \( r(t) \) is actually the error signal amplified by the PID controller. Usually, the desired set point \( u^* \) is a given constant and the dimensionless error is defined as \( e = 1 - \frac{u(t)}{u^*} \).

For a control valve, \( u(t) \) could be either of the inlet pressure head \( H_1(t) \), outlet pressure head \( H_2(t) \), or the flow passing through the valve \( Q(t) \). In a physical system, the values of these control variables are being continuously measured by sensors; while in numerical counterpart, the values of these variables are simulated step by step.

The control law expressed by equation (4) is general for all types of PID-controllers. It is straightforward to set the parameter \( T_d \) to zero for a PI-controller; furthermore, for a controller that has proportional part only (i.e., a P-controller), \( T_i \) is given a large value. Besides, for a series-structured controller with given parameters, the parameters for corresponding parallel type can always be obtained by the relationships
between these two controller structures. On the other hand, given the parameters for a parallel type controller, it is not always possible to obtain the corresponding parameters for the series type (Tan. et al. 1999). Therefore, equation (4) for parallel type PID controllers will work for series type as well by transforming the given parameters to those for the corresponding parallel type.

Depending on the power source of the actuator (pneumatic, electric or hydraulic) and the internal design of the valve and controller, the operational equation of the PID controller could be established corresponding for each specified process variable. For the control of pressure at the valve inlet, $H_1$, the output signal (the amplified error) of the controller is set equal to the rate of valve flow reduction (Hopkins 1998); that is,

$$r(t) = K_C (e + \frac{1}{T_i} \int_0^t e \, dt + T_d \frac{de}{dt}) = -\frac{dQ}{dt}$$

(5)

here $e = 1 - \frac{H_1(t)}{H_1^*}$. The hydraulic implication of the above control action can be explained as follows: when pressure $H_1(t)$ at the inlet of the valve begins to exceed the set point $H_1^*$ ($e < 0$), the valve would open slightly to discharge the excess water volume ($\frac{dQ}{dt} > 0$). By contrast, if the pressure $H_1(t)$ at the valve inlet begins to decay below the set point $H_1^*$ ($e > 0$), the valve would throttle and reduce the discharge ($\frac{dQ}{dt} < 0$). With a PID controller, the automatic adjustment of the valve opening would be smooth and continuous.
Similarly, for the pressure control at the outlet of the valve $H_2$, the output signal of the controller would be set equal to the rate of valve flow increase ($+\frac{dQ}{dt}$). That is, the minus sign is removed from the right-hand side of equation (5) and one would use $e = 1 - \frac{H_2(t)}{H_2^*}$ instead.

For the control of valve flow rate $Q$, the signal of control error is set equal to the change of the head difference across the valve, $H$, and this change is based on the initial steady state, that is,

$$r(t) = K_c (e + \frac{1}{T_i} \int_0^t edt + T_d \frac{de}{dt}) = H_0 - H(t)$$

$$= [H_{10} - H_{20}] - [H_1(t) - H_2(t)]$$

(6)

here $e = 1 - \frac{Q(t)}{Q^*}$.

In sum, for a local/conventional PID control valve, two MOC equations, the valve discharge equation and the PID controller equation constitute the mathematical model of the boundary condition in the pipe network. Therefore, the four unknown variables at valve boundary ($\tau, H_1, H_2, Q$) can be numerically resolved using the finite difference method.
4. Consideration and Model of “Non-Reflective” Valve Opening

Based on the developed mathematical model for a local PID control valve, the physical consideration and mathematical model to search for a “non-reflective” boundary by using remote sensing and PID control valve is elucidated in this section.

Theoretically, a remote sensor could be located either upstream or downstream of the control valve wherever an incident pressure surge occurs, since the pressure surge will usually propagate towards both directions. However, this research focuses on the case of upstream incident surge, and the main idea is illustrated by a simple system shown in Figure 5.

In this system, it is assumed that a series of pressure surges, \( H_R(t) \), (say, that represent periodic waves following a sinusoidal law) occur at an upstream reservoir due to a certain disturbance, where a sensor is installed and linked to a PID control valve at downstream reservoir. The pressure waves would then propagate at speed \( a \) towards the downstream end along the pipeline. The wave would arrive at the valve inlet within \( L/a \) seconds. To limit the wave reflection from the valve, the PID controller adjusts the valve opening continuously and accurately to "absorb" the upcoming incident wave when it passing through the valve (i.e., to dissipate the wave energy at the valve). As a result, at any instant time \( t \), the pressure at the valve inlet \( H_1(t) \) should maintain the same magnitude as the upcoming wave when it reaches the valve. In other words, the set point of valve inlet pressure, \( H_1^* \), would be equivalent to the “successor” of the upstream reservoir pressure at a \( L/a \) time ahead, i.e., \( H_R(t-L/a) \). Since the upcoming pressure
changes with time, the set point, $H_1^*$, must change with time as well, which is so-called **time-variable or dynamic set point**, $H_1^*(t)$.

The pressure at the valve inlet $H_1(t)$ will be tracked in this “non-reflective” boundary model, correspondingly, the mathematical model for the local PID control valve with $H_1$ control case, including the extended MOC equations (1) and (2), the valve discharge equation (3) and the PID controller equation (5), is applicable. However, there is a key difference in the controller equation; in particular, the control error $e$ in “non-reflective” boundary model is the deviation of instant pressure head $H_1(t)$ from its dynamic set point $H_1^*(t)$, that is, $e = 1 - \frac{H_1(t)}{H_1^*(t)}$. Instead of a given constant for the set point in the local control valve, the set point $H_1^*(t)$ in this case is an unknown variable (transmission of $H_R$), dependent of the remote pressure waves and pipe system features, which could significantly complicate the discretization of the governing equations and their numerical solution. Yet, if $H_1^*(t)$ could be predicted at any instant time $t$, then the PID controller will send the actuator a series of “commands” (the error signals) to adjust the valve opening continuously to achieve $H_1(t) = H_1^*(t)$. Therefore, the only remaining issue is how to determine the dynamic set point, $H_1^*(t)$, according to the remotely sensed pressure $H_R(t)$?

For a frictionless pipeline without reflections from the downstream boundary, the upstream pressure wave $H_R(t)$ would have no change when it propagates to the front of valve in $L/a$ seconds. Thus, at any instant, the set point for the pressure at the valve inlet is equivalent to the pressure at the remote sensor that has been recorded at prior time of $L/a$. Mathematically, we have:
However, for a more realistic pipeline having friction resistance, the magnitude of a pressure wave decays somewhat as it propagates downstream. The key and challenging question is how to transform transient pressures between the two ends of the pipeline with friction. Since no analytic solutions are available, the set point must be estimated based on the remotely measured pressure waves and pipe system features. In this research, a relative Friction Decay Rate ($F_{DR}$) is introduced based on the hydraulic grade line at initial steady state, which is defined as the ratio of pressure heads at the valve inlet ($H_{10}$) and at the remote sensor ($H_{R0}$):

\[
F_{DR} = \frac{H_{10}}{H_{R0}} \tag{8}
\]

Certainly, $F_{DR}$ is system specific but constant for a particular system with certain initial hydraulic condition. Using the $F_{DR}$ defined in equation (8), the magnitude of pressure wave when it arrives at the valve could be largely estimated:

\[
H_{i}^*(t) = F_{DR} \times H_{R}(t - L/a) \tag{9}
\]

Note that equations (8) and (9) are consistent with and valid for frictionless pipelines as well. When the pipeline friction is negligible we have $H_{R0} = H_{10}$ and $F_{DR} = 1$, and thus equation (9) reduces to equation (7). Therefore, for both frictional and frictionless pipelines, the set point in controller equation (5) could be dynamically estimated using equations (8) and (9). Interestingly, for those cases with an initially static state (i.e., the initial flow $Q_0 = 0$ in the system), there is no initial headloss along
the pipeline no matter the pipeline is frictional or frictionless, so we also have $H_{R0} = H_{10}$ and $F_{DR} = 1$, and equation (9) still reduces to equation (7).

Through this FDR compromise, we are actually using the steady state FDR to estimate the frictional decay under transient states. Fortunately, the examples using this model have sufficiently shown the potential of PID control valve with remote pressure sensor in limiting the pressure oscillations and resonance. As better pressure control would arise from a better estimate for the dynamic set point, $H_1^*(t)$; but this value is challenging to find since there is no universally appropriate estimate for transient pressure decay rate. These difficulties arise from the complexity of the friction term and wave interference in the momentum equation of transient flow, which is not only dependent of the pipe roughness, other system properties and the hydraulic conditions, but also relevant to the direction and frequency of the propagating waves (Suzuki et al. 1991; Tirkha 1975; Vardy et al. 1993; Vardy and Brown 2003; Vardy and Brown 1995; and Zielke 1986).

Furthermore, it is understood that the pressure at the remote location for the $L/a$ second earlier, $H_R(t-L/a)$, could be always retrieved at any instant time $t$ in both physical and numerical systems. In the physical system, the value of $H_R(t-L/a)$ was measured and recorded at the remote sensor, and then sent instantly to the controller by wireless or cable transmission at any time $t$. Thus, at least two pressure sensors are required in the system, one is to measure the control variable $H_1(t)$ at the valve and another is to measure remotely to obtain the dynamic set point $H_1^*(t)$. While in the numerical counterpart, the pressure at the remote sensor node for $L/a$ second time ahead, $H_R(t-L/a)$, was already simulated and stored, which could obviously be retrieved at any instant time.
Finally, it is noticed that the entrapped air will significantly lower the wave speed, which, in turn, will affect on the value of dynamic set point $H_1^*(t)$ according to Equation (9). The simulation results may be thus deteriorated by the incorrectly assumed wavespeed and time delay $L/a$. Therefore, the system to simulate must be carefully calibrated in advance by the measurements in the real system, air entry needs to be monitored and prevented during the wave travelling, and thus the wavespeed is able to largely determined beforehand for the specific system. As it so often does, the presence of air in an otherwise liquid pipeline threatens to cause much mischief to both operators and designers.

5. Simulations and Examples

5.1 Examples with Non-Zero Initial Flow

The same system, as shown in Figure 5, is studied to illustrate the application of current mathematical model. In this system, a long horizontal pipeline links two constant head reservoirs. At the entrance of downstream reservoir, there is a valve with constant $E_S = 7 \text{ m}^2\text{s}^{-1.5}$ and initial opening $\tau_0 = 0.1$. The upstream reservoir has 30 m water head initially ($H_{R0} = 30 \text{ m}$) and downstream reservoir has a constant head of 15 m. The water level at upstream reservoir starts to fluctuate sinusoidally, which induces transient events in the system. The amplitude of oscillatory wave is 2 m; the cyclic period is 10 s, which equals $2L/a$, so that resonance would be expected to take place. The instant pressure at the upstream reservoir can be described as:

$$H_R(t) = H_{R0} + 2 \sin(2\pi \ t / 10)$$

(10)
In the first example, it is assumed the pipeline is frictionless, i.e., the Darcy-Weisbach friction factor \( f = 0 \). At initial steady state, the hydraulic head at the valve inlet equals the reservoir head, that is, \( H_{10} = H_{R0} = 30 \) m, and the flow rate in the system and passing through the valve is \( Q_0 = 2.71 \) m\(^3\)/s.

If the downstream valve remains at its initial opening \( \tau_0 = 0.1 \) (i.e., fixed orifice), the oscillations of upstream pressure (i.e., incidental pressure wave, or forced vibration) will cause hydraulic resonance and pressure amplification in the middle part of the pipeline. Figure 6 shows the development of the steady-oscillatory flow in the pipeline; the red/solid line represents the pressure oscillations at the middle-point of the pipeline and the black/dashed line represents the pressure oscillations at the valve inlet. There is a \( L/2a \) time difference (i.e., 1/8 phase difference) between these two pressure waves, and the amplitudes of both waves are initially small and grow gradually until they finally stabilize at a resonance condition (within 300 s). This mode shape of the pressure waves can be understood and explained as follows: in this system, the upstream incidental wave will propagate along the pipeline and reach the valve inlet in 5 s (\( L/a = 5 \) s), and thus it will take 7.5 s for the first peak of the sinusoidal wave (with the oscillatory period 10 s) to arrive at the valve inlet. During the time period 5 to 7.5 s, the pressure at the valve inlet, indeed at any internal pipe node, is the superposition of the incident wave and the reflected wave from the fixed valve at downstream reservoir. Since the reflection at downstream boundary in this case is negative, the first wave peak is reduced when it arrives at downstream valve. However, the pressure waves with different amplitudes would continuously proceed and be reflected. The result of superposition increases until the maximum amplitude reached and the steady-oscillatory flow condition developed in
the system. This simulation result with fixed valve opening is also represented by the red/dashed lines in Figure 7. The value of the steady amplitude at each position of pipeline depends on the system frequency and resistance characteristics. As matter of fact, this phenomenon resulting from the forced vibration of upstream pressure (with fixed downstream valve opening) is equivalent to the responses caused by the periodic valve motion at downstream end (while keeping the upstream reservoir head constant), as summarized in Figure 8.2 on page 204 of Chaudhry (1979).

To eliminate or limit the reflection and superposition of pressure waves in the pipeline, a sensor is installed at the upstream node, and a PID controller positioned at the downstream valve. The constants of PID controller are taken as $K_C = 250$, $T_i = 0.002$ s and $T_d = 0.5$ s. The total simulation time is run for 800 s, and the time step is $1/100$ s that is sufficiently small to look into the detailed valve responses.

The developed remote control model for frictionless pipeline is used in this example to simulate the system responses to the oscillations of pressure head at the upstream reservoir. Figure 7 compares the system responses for the responsive PID control valve (black/solid lines) and the conventional valve with fixed orifice (red/dashed lines). The black/solid lines in Figure 7 (a) show that the maximum and minimum pressures along the pipeline remain the same as those introduced at upstream reservoir, demonstrating no wave reflection or resonance ever occurred in the system by using remote sensor and PID control valve. In Figure 7 (c), the black/solid curve shows that the opening of PID control valve is adjusted periodically in response to the oscillating incident pressure waves, as expected, which exactly eliminates the wave reflections at
the valve (by exciting a contrary wave) and thus remains the same mode shape of pressure oscillation as of the upstream incident wave, see the black/solid $H_1$ waves in Figure 7 (b).

By contrast, if the valve opening is fixed at the initial size ($\tau = 0.1$) as the straight red/dashed line in Figure 7 (c), the amplitude of pressure waves at the inlet of fixed orifice reduces at the beginning and grows gradually until finally stabilized at certain value because of resistance of the valve, as shown in the red/dashed $H_1$ wave in Figure 7 (b). Correspondingly, the pressure waves are reflected and superposed in the pipeline, resulting in the increased envelope of maximum and minimum pressures along the pipeline, see the red/dashed curves in Figure 7 (a).

In the second example, the friction factor $f = 0.012$ is given for the pipeline. At the initial steady state, the hydraulic grade line declines along the pipe, the pressure at the valve inlet is $H_{10} = 19.37$ m, and the flow in the system and passing through the valve $Q_0 = 1.46$ m$^3$/s. The same parameters of PID controller, total simulation time and time step as in the first example are used here.

To simulate the system responses to the oscillations of pressure head at upstream reservoir, the frictional decay rate $F_{DR}$ defined in equation (8) and the dynamic set point estimated by equation (9) are applied. The simulated PID control results are compared with the corresponding conditions for fixed-opening-valve case. As shown in Figure 8, the reflection is not completely eliminated because of the roughly estimated $F_{DR}$ in transient state. However, the simulation results converge to a steady oscillation
flow within 400 s, and the comparison in Figure 8 (a) shows that the maximum and minimum pressure envelope using the PID control (solid/black curves) is smaller than the case of fixed-opening-valve (dashed/red curves), which demonstrates the reflections and resonance of pressure waves are constrained by using the remote sensor and PID control valve. The smaller the initial flow in the system, the more reduction in the magnitude of pressure amplitude envelopes, and this point can be demonstrated by the following example with zero initial flow. Figure 8 (c) shows that the opening of the PID control valve is adjusted periodically in response to the proceeding waves, which, as shown in Figure 8 (b), creates a steady oscillation at the valve inlet with the same decayed amplitude as initial steady state. Yet, for the case of fixed-opening-valve, the amplitude of pressure waves at the valve inlet varies with time, reduced at the beginning and increasing gradually until a steady-oscillatory flow developed over about 300 s.

5.2 Examples with Zero Initial Flow (Static Initial State)

In the system sketched in Figure 5, there is no flow in the system and the initial valve opening is of no consequence if the constant heads (30 m) are remained at both upstream and downstream reservoirs. Yet, if a sinusoidal oscillation of head at the upstream reservoir is initiated, a flow in the pipeline will be created, and the flow direction shifts as the head oscillates around the original constant level. At any specific point of the pipeline (e.g., at the valve), the magnitude of oscillatory flow is small and the average value with time is zero. The $f$ value by itself seems hardly influence the wave reflection. Therefore, for both cases with frictionless and frictional pipeline, the wave reflection and resonance can be completely eliminated and the “non-reflective” boundary achieved at
the downstream valve, if the remote sensing and PID controlled valve are implemented in the system.

Figure 9 shows the simulation results including the friction factor of 0.012. Without the flow (and thus without resistance from flow), the resonance would be stabilized in around 600 s for the fixed valve opening case and the range of the max./min. pressures are much larger than that with non-zero flow case. While with responsive PID control valve, there is no wave reflection and resonance at all in the system, and the oscillations remain steady at any point and at any time, as the same as the upstream incident pressure oscillations.

6. Frequency Analysis and “Non-Reflective” Boundary Verification

Based on the time domain transient analysis using method of characteristics (MOC), the numerical model for “non-reflective” boundary design has been developed in previous sections. However, for a periodic oscillation originating at a remote location, transient analysis in the frequency domain is a more practical and efficient way to reveal the oscillatory conditions in the fluid system.

In this section, oscillation is introduced through various harmonics that might occur in the pipe system. Then, the system responses to the forced vibrations (upstream pressure oscillations) with different frequencies and the applicability of developed “non-reflective” model are checked. After that, the steady “non-reflective” boundary conditions for the frictionless pipeline system, obtained by the traditional MOC, would be verified using the method of hydraulic impedance in the frequency domain.
6.1 System Responses to Pressure Oscillations with Various Frequencies

Unexpected resonance could be destructive in practical hydraulic systems. The consequences of resonance in fluid systems range from objectionable operating conditions, such as instability, noise, and vibration, to fatal damage of system elements overstressed during severe pressure oscillations. Thus, the phenomenon of hydraulic resonance should be predicted and prevented.

In the example discussed in previous section, the incident pressure oscillation at upstream reservoir is one type of forced excitation. The fundamental period of pipeline system $T_0 = 4L/a = 20 \text{ s}$ (i.e., natural frequency is $1/20 \text{ Hz}$) and the given period of forced excitation $T = 10 \text{ s}$ (the forcing frequency is $1/10 \text{ Hz}$), and the system responses to this forced excitation are shown in Figure 7 (a) and Figure 8 (a) for frictionless and frictional pipeline, respectively. For a fixed orifice at downstream reservoir, the system responses demonstrate the characters of second harmonics, and the maximum amplitude of pressure oscillation, occurred at the middle point of the pipeline, is about 12 and 9 times as large as the incident pressure oscillation, respectively, for frictionless and frictional cases. Now, what if we change the frequency of the forced excitation? Could we adjust downstream valve opening to eliminate the potential resonance in the system using the developed “non-reflective” boundary design?

**Frictionless pipeline system with non-zero initial flow.** In the system with the fixed valve opening at downstream reservoir (Figure 5), even order of harmonics exist, as shown in Figure 10 (b), (d) and (f). The even harmonics indicates that the reflective characteristic at downstream orifice is similar to a reservoir (negative reflection) (Wylie
et al. 1993 and Chaudhry 1987). This can be verified by comparing the hydraulic impedance of the fixed valve with the characteristic impedance of the system. Hydraulic impedance in a fluid system is defined as the ratio of the complex head to the complex discharge at a particular point in the system (Wylie et al. 1993).

For the frictionless pipeline system as shown in Figure 5 (reservoir-pipeline-orifice at reservoir), the characteristic impedance of fluid system is calculated as follows:

\[
Z_c = \frac{a}{gA} = \frac{1000 \text{ m/s}}{9.81 \text{ m/s}^2 \times 3.1416 \times 1^2/4 \text{ m}^2} \approx 130 \text{ s/m}^2
\]  

(11)

While for the fixed orifice or valve, the valve equation (3) can be written as:

\[
\overline{Q} = \tau E_S \sqrt{\overline{H}_0}
\]  

(12)

in which \(\overline{H}_0\) is the head drop across the valve for the mean flow \(\overline{Q}\). From the initial steady state, we have: \(\tau = 0.1, \ E_s = 7 \text{ m}^{2.5}/\text{s}, \ \overline{H}_0 = 15 \text{ m}, \ \overline{Q} = 2.711 \text{ m}^3/\text{s}\). So the hydraulic impedance of this fixed valve:

\[
Z_v = \frac{H_v}{Q_v} = \frac{2\overline{H}_0}{\overline{Q}} \approx 11 \text{ s/m}^2
\]  

(13)

here \(H_v\) and \(Q_v\) are the complex head and flow at the oscillatory valve, respectively. Thus, we have \(Z_v < Z_c\), this is the condition that even harmonics occurs in the system. On the contrary, if \(Z_v > Z_c\), the odd harmonics could occur, which indicates the orifice would provide a response similar to a dead-end. If we adjust the valve opening to make \(Z_v = Z_c\), then the orifice becomes “non-reflective”.

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When the cyclic period of the forced vibration is given as fractional part of the fundamental period \( T_0 = 4L/a = 20 \) s, the different orders of harmonics and different mode shapes of pressure waves would occur in the pipe system. However, the frequency change doesn’t affect the amplitude of each harmonics if the amplitude of incidental pressure oscillation remains the same. The order of harmonics equals the system fundamental period \( T_0 \) divided by the period of forced vibration \( T \). If we give even number of \( T_0/T \) (i.e., \( T = 10 \) s, 5 s, 3.3 s, 2.5 s, …), the excessive energy influx to the system during oscillatory flow leads to resonance, as shown in Figure 10 (b), (d) and (f).

In an ideal lossless system (for this frictionless pipeline system the only energy dissipation occurs at the downstream valve), there is generally no energy transmission in steady-oscillatory flow, although alternating energy conversion between kinetic energy and pressure energy may occur. With terminal wave reflection, steady-oscillatory motion shows a combination of forward and reflected waves which results in a standing wave. Within the standing-wave pattern, energy is converted from pressure energy to kinetic energy, then back to pressure energy, and so on. If we give odd number of \( T_0/T \) (i.e., \( T = 20 \) s, 6.67 s and 4 s, …), as shown in Figure 10 (a), (c) and (e), the mode shape of the pressure waves is quite different from the even harmonics, the energy dissipated gradually and resonance does not occur in the system.

It is not surprising to find that the resonance with different orders of harmonics and amplification of pressure head can be completely eliminated in frictionless pipeline by designing a “non-reflective” boundary. The simulated results are all the same as the black/dotted line in Figure 7 (a), no matter how the period of the forced vibration \( T \) changes. However, it is noticed that for the higher frequency forced oscillations (smaller
$T$), the PID integral and derivative parameters ($T_i$ and $T_d$) require smaller values to obtain the precise adjustment of “non-reflective” valve opening.

**Frictional pipeline system with non-zero initial flow.** For the frictional pipeline case, we have a similar finding regarding even harmonics when we change the period of the forced vibration. The application of remote sensor and PID control valve cannot completely eliminate the reflections and resonance if the initial flow is not zero, but the amplitude of the pressure waves is significantly reduced for each harmonics, as shown Figure 11. In this Figure, the different responses to the forced oscillations at the upstream end are compared for the system with a fixed-opening-valve and responsive PID valve at the downstream end of pipeline.

**6.2 “Non-Reflective” Boundary Verification using Hydraulic Impedance Method**

From the viewpoint of frequency domain, the automatically adjustable PID valve creates an artificial excitation and the consequence of this designed valve-oscillation would exactly cancel out the effect of incidental pressure oscillation at the upstream reservoir. For the frictionless pipeline system, we have verified the condition of a “non-reflective” boundary, that is, $Z_x = Z_c$, for the developed steady-oscillatory flow (i.e., after 300 s of PID valve adjustment when the amplitude of the pressure waves in the pipeline stabilized). This steady valve-oscillating condition has been obtained from the numerical simulation in the time domain, which uses the “non-reflective” boundary model developed in the previous section.
For the frictionless pipeline system, the characteristic impedance has been calculated as equation (11). For an oscillating valve, the hydraulic impedance at the upstream side of the valve can be calculated by (Wylie et al. 1993)

\[
Z_V = \frac{H_V}{Q_V} = \frac{2\overline{H}_0}{\bar{Q}} - \frac{2\overline{H}_0}{\bar{T}_V}\frac{T_V}{Q_V}
\]  

(14)

We already know \(\bar{T} = 0.1, \overline{H}_0 = 15\) m, \(\bar{Q} = 2.711\) m\(^3\)/s, and here \(H_V, Q_V\) and \(T_V\) are the complex head, flow and opening at the oscillatory valve, respectively.

From numerical simulation in the time domain, as the results shown in Figure 7, we found the maximum valve opening \(\tau_{\text{max}} = 0.107\) corresponding to the minimum valve flow \(Q_{\text{min}} = 2.695\) m\(^3\)/s), and minimum valve inlet head \(H_{1\text{min}} = 28.0\) m. On the contrary, the minimum valve opening \(\tau_{\text{min}} = 0.094\), corresponds to the maximum flow \(Q_{\text{max}} = 2.726\) m\(^3\)/s) and maximum valve inlet head \(H_{1\text{max}} = 32.0\) m). Here, the interesting parts of pressure, flow and valve opening time-histories are the pure oscillatory parts of them; and a half of the difference between the maximum and minimum values is used as the amplitude in the following calculations. In other words, we have complex hydraulic values:

\[
H_V = 2.0 \sin \left[ \frac{2\pi (t + L/\alpha)}{10} \right] = 2.0 \sin \left[ \frac{2\pi t}{10} + \pi \right]
\]  

(15)

\[
Q_V = 0.0155 \sin \left[ \frac{2\pi (t + L/\alpha)}{10} \right] = 0.0155 \sin \left[ \frac{2\pi t}{10} + \pi \right]
\]  

(16)
\[ T_V = 0.0065 \sin \left( \frac{2\pi t}{10} \right) \]  

(17)

It is noticed that there is a phase difference of \( (2\pi L/a)/10 \) between the oscillation of valve opening \((T_V)\) and the oscillations of valve flow \((Q_V)\) and inlet head \((H_V)\). So, we can also calculate 

\[ Z_V = \frac{H_V}{Q_V} \approx 129 \text{ s/m}^2, \quad \text{or} \quad Z_V = \frac{2H_0}{Q} - \frac{2H_0}{\tau} \frac{T_V}{Q_V} \approx 137 \text{ s/m}^2 \]  

(from equations (16) and (17), we know \(T_V\) and \(Q_V\) have opposite sign; the numerical error is acceptable due to only 3 digits of the simulation results recorded). Therefore, the “non-reflective” boundary condition \(Z_V \approx Z_c = 130 \text{ s/m}^2\) has been verified.

In the case without initial flow but the pipeline with friction, as shown in Figure 8, we obtained the same pressure and flow oscillations at the valve as described in equations (15) and (16), and thus 

\[ Z_V = \frac{H_V}{Q_V} \approx Z_c = 130 \text{ s/m}^2 \]  

can be verified. Moreover, by changing the amplitude of the incident pressure waves at the upstream reservoir, the amplitude of induced flow oscillations would change proportionally, so the value of 

\[ Z_V = \frac{H_V}{Q_V} \]  

remains near 130 and “non-reflective” boundary condition would be always achieved for static initial state cases.

To further verify this law of “non-reflective” boundary condition, the wave speed of pipeline is reduced to 500 m/s, so the system characteristic impedance also reduces to 

\[ Z_c = \frac{a}{gA} \approx 65 \text{ s/m}^2. \]  

The “non-reflective” boundary condition, \(Z_V = Z_c\), could be verified as well by the corresponding numerical simulation results in the time domain.
7. Tuning PID Controller

The final tuning of the parameters \((K_C, T_i \text{ and } T_d)\) for a PID controller would be important during the commissioning stage of the system. Similar to the trial and error method used for a physical system, the numerical model could provide a tool for preliminary selection of these parameters. To better understand how the variation of each parameter affects on the system control results, a sort of sensitivity analysis for three controller parameters is performed here, using the same system shown in Figure 4 with frictionless pipeline as aforementioned, and the comparative results are summarized in Figure 12.

Proper selection of controller parameters means finding a compromise between the requirement for fast control and the need of stable control. More specifically, with increases in the proportional gain \((K_C)\), the speed of control increases but the stability of control reduces. Figure 12 (a) shows that given the same simulation time period \((200 \text{ s})\), the reduction of \(K_C\) (slower control) enlarges the maximum and minimum pressure envelope. Clearly, \(K_C\) must be greater than a certain value to effectively control the system. In this example, a \(K_C\) of about 50, or greater, is needed.

The tendencies for the variation of integral time \((T_i)\) are opposite to \(K_C\). With \(T_i\) increases, the speed of control reduces while the stability of control increases. Figure 12 (b) shows that the maximum and minimum pressure envelope expands with the increase of integral time \(T_i\) (slower control), and \(T_i \leq 0.001 \text{ s}\) is required in this system.
The derivative part produces both faster and more stable control when $T_d$ increases. However, this is only true up to a certain limit and if the signal is sufficiently free of noise (calculation error is a kind of noise in numerical system). If $T_d$ rises above this limit it will result in reduced stability of control. As we know that the function of the derivative part is to estimate the change in the control a time $T_d$ ahead. This estimate will naturally be poor for large values of $T_d$. Another consideration is the noise and other disturbances. The noise is amplified to a greater extent when $T_d$ increases, and thus it is often the noise that sets the upper limit for the magnitude of $T_d$. The above theoretical analysis could be verified by the numerical simulations in Figure 12 (c), which shows that the control stability is better when $T_d = 0.5$ s (dashed lines) than that when $T_d = 0.005$ s (dot-dash lines). However, when $T_d = 5$ s the stability of control is poor.

In addition, the simulations also show that for high frequency oscillatory flow (e.g., 1 s of the period of incidental pressure wave), the smaller values for both the integral time constant $T_i$ and derivative time constant $T_d$ are required.

The stability and speed of control process are associated with the parameters of controller. The mathematical model and numerical simulation are useful for selection and tuning of the controller parameters, which could save time and cost in commissioning of the physical system.

8. Conclusion and Discussion

The creation of “non-reflective” (or “semi-reflective”) boundaries through remote sensing and PID control valve is a new concept to more accurately limit dead-end
reflection and resonance in pipelines. This idea is explored here by the means of numerical simulation, and a considerable potential for transient protection has been demonstrated in the numerical examples. As shown, wave reflection and resonance can possibly be eliminated for frictionless pipelines or the pipelines with static initial states; while for the pipeline with some friction, the pressure waves’ reflection at the valve and superposition within the pipeline can be effectively limited. Moreover, the feature of even order of harmonics as well as “non-reflective” boundary conditions during steady oscillation, obtained through time domain transient analysis, are verified by hydraulic impedance analysis in the frequency domain. Certainly dead-end branches are a common component in pipe networks; at such locations, dead-end reflection may cause unexpected high pressures when system experiences transients, and such locations might readily be improved with appropriate thought to tailored wave reflections such as those described in this paper.

However, complications and challenges will inevitably arise when the model of “non-reflective” boundary is applied in practice. First, the model is developed based on a single pipeline system with the remote sensor at upstream end and PID control valve at downstream end. It is feasible to apply the model for a branched pipeline in a complex system with remote sensor at the junction and a terminal valve at the branch-end, but further application to a pipe loop or an arbitrary pipeline with other components between the two ends would be significantly challenging, since the primary pressure wave would be reflected, refracted, or attenuated by those components and thus the theoretical estimate of pressure set point become almost impossible. Besides, in real systems, the uncertainties (including entrapped air effect or frequency-relevance) in the magnitudes of
waves speed, pipe friction factor and pressure decay rate at transient state may cause significant over- or under-estimate of pressure set point and thus require for careful system calibrations in advance. The upstream pressure disturbance may not be a steady oscillation as in the real systems, and then the reflections at the upstream reservoir would further complicate the “non-reflective” valve opening control. Thus, at present the design of “non-reflective” boundary, though promising, calls for considerable further research as and likely will require cooperation with device manufacturers.

References


*Proc., International Conference on Unsteady Flow and Fluid Transients*, Durham, United Kingdom, 343-352.


Figure 1 Scheme of a branched system and initial pressure head

Note: Pipe length AB = BC = CD = DE = CF = DG = 1000 m, friction factor $f = 0.012$, the diameter of main pipeline $D = 1$ m, and the diameter of branches $d = 0.5$ m. Initially, the terminal control valves at the end of branches are fully closed. If the pipes do not leak and have been open to the reservoirs for some time, no flow will occur in the system and the head will be uniformly 100 m as found in the reservoirs. Following Boulos et al. (2006), suppose the second section of the main pipeline (from B to C) is pressurized to a uniform value of 130 m.
(a) Transient response in the system with dead-end branches

(b) Transient response in the system with 10% orifices at branch ends

(c) Transient response in the system with 0.35% orifices at branch ends

Figure 2 Comparison of transient responses to dead-ends and small orifices
Figure 3 Traveling pulse waves and “tailored” valve reflections
Figure 4 PID-controlled valve in a pipe network
Figure 5 System scheme and initial steady state

Note: pipe length $L = 5000$ m, diameter $D = 1.0$ m, and wave speed $a = 1000$ m/s

Figure 6 Development of steady-oscillatory flow in frictionless pipeline with fixed-valve-opening at downstream, due to the upstream pressure oscillations
(a) Maximum and minimum pressure head envelopes

(b) Inlet pressure wave varying with time
(c) Valve opening varying with time

Figure 7 Responsive PID control valve vs. fixed-opening-valve in frictionless pipeline ($f=0$)

(a) Maximum and minimum pressure head envelopes
(b) Inlet pressure wave varying with time

(c) Valve opening varying with time

Figure 8 Responsive PID control valve vs. fixed-opening-valve in frictional pipeline ($f=0.012$)
(a) Maximum and minimum pressure head envelopes

(b) Inlet pressure wave and flow at the PID valve varying with time

Figure 9 Responsive PID control valve vs. fixed-opening-valve for static initial states with frictional pipeline ($f=0.012$)
(a) No resonance ($T_0/T = 1$)

(b) 2\textsuperscript{nd} order of harmonics ($T_0/T = 2$)

(c) No resonance ($T_0/T = 3$)
(d) 4\textsuperscript{th} order of harmonics ($T_0/T = 4$)

(e) No resonance ($T_0/T = 5$)

(f) 8\textsuperscript{th} order of harmonics ($T_0/T = 8$)

Figure 10 System responses to forced pressure oscillation with various frequencies in a frictionless pipeline ($f=0$) (Fixed-valve-opening)
(a) $t=10 \text{ s}$

(b) $t=5 \text{ s}$
Figure 11  System responses to forced pressure oscillation with various frequencies in a frictional pipeline ($f=0.012$) (Fixed valve vs. responsive PID valve)
(a) Max./Min. pressure envelopes expanding with reduction of proportional gain $K_C$

$(T_i = 0.001 \text{ s}, \ T_d = 0.5 \text{ s})$

(b) Max./Min. pressure envelopes expanding with increase of integral time $T_i$

$(K_C = 250, \ T_d = 0.5 \text{ s})$
(c) Max./Min. pressure envelopes varying with different derivative time Td

$$(K_C = 250, \ T_i = 0.02 \ s)$$

**Figure 12** Effect of controller parameters on system pressure control
Figure Captions

Figure 1 Scheme of a branched system and initial pressure head

Figure 2 Comparison of transient responses to dead-ends and small orifices

Figure 3 Traveling pulse waves and “tailored” valve reflections

Figure 4 PID-controlled valve in a pipe network

Figure 5 Examples: System scheme and initial steady state

Figure 6 Development of steady-oscillatory flow in frictionless pipeline with fixed-valve-opening at downstream, due to the upstream pressure oscillations

Figure 7 Responsive PID control valve vs. fixed-opening-valve in frictionless pipeline (f=0)

Figure 8 Responsive PID control valve vs. fixed-opening-valve in frictional pipeline (f=0.012)

Figure 9 Responsive PID control valve vs. fixed-opening-valve for static initial states with frictional pipeline (f=0.012)

Figure 10 System responses to forced pressure oscillation with various frequencies in a frictionless pipeline (f=0)

Figure 11 System responses to forced pressure oscillation with various frequencies in a frictional pipeline (f=0.012) (Fixed valve vs. responsive PID control valve)

Figure 12 Effect of controller parameters on system pressure control
Notation

\( a \) – wave speed

\( D \) – diameter of pipe cross section

\( e \) – control error or deviation of the process variable \( u(t) \) from its set point \( u^* \);

dimensionless error is defined as \( e = 1 - \frac{u(t)}{u^*} \).

\( E_S \) – a valve size parameter determined by the energy dissipation potential of the valve.

\[
E_S = \frac{Q_0}{\sqrt{\Delta H_0}} \quad \text{(in SI unit of } m^{2.5}/s, \text{ but valve manufacturers typically report } E_S \text{ in units of } \text{USgpm}/\sqrt{\text{psi}}).\]

\( f \) – Darcy-Weisbach friction factor

\( F_{DR} \) – Relative Friction Decay Rate, which is defined as the ratio of pressure heads at the valve inlet (\( H_{10} \)) and at the remote sensor (\( H_{R0} \)):

\[
F_{DR} = \frac{H_{10}}{H_{R0}}.
\]

\( H \) or \( H(t) \) – instant pressure head

\( H_1 \) or \( H_1(t) \) – instant pressure head at valve inlet

\( H_{10} \) – the initial pressure head at valve inlet

\( H_1^*(t) \) – the dynamic set point of the pressure head at valve inlet \( H_1(t) \)

\( H_2 \) or \( H_2(t) \) – instant pressure head at valve outlet

\( H_R \) or \( H_R(t) \) – instant pressure head at reservoir

\( H_{R0} \) – the initial pressure head at reservoir

\( K_C \) – the proportional gain of controller

\( L \) – pipe length

\( Q \) or \( Q(t) \) – instant flow in pipeline or passing through a valve

\( Q_0 \) – initial steady state flow in pipe system or passing through a valve

\( \Delta H_0 \) – pressure head difference across a valve at initial steady state

\( \Delta H \) or \( \Delta H(t) \) – instant pressure head difference across a valve

\( t \) – time

\( T_i \) – integral time constant of controller

\( T_d \) – derivative time constant of controller

\( u(t) \) – process variable to be controlled

\( u^* \) – set point or desired value of process variable

\( \tau \) – Dimensionless valve opening